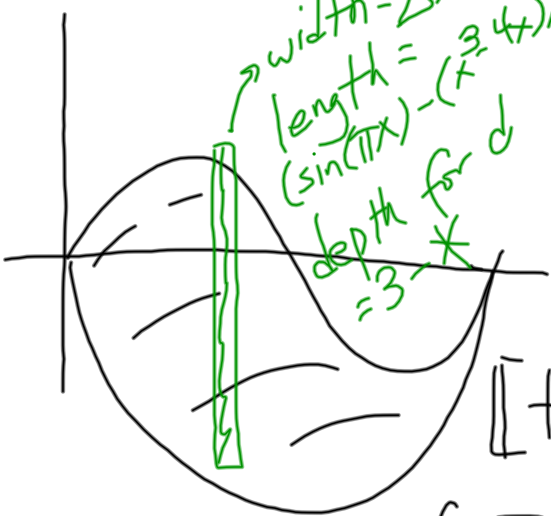
1)



a) area of R =

$$\int_0^2 \sin(\pi x) - (x^3 - 4x)\,dx =$$

[type in calculator

fnInt(sin(πx) - (x³-4x), x, 0, 2)

- make sure radian mode]

=

b) solve

$x^3 - 4x = -2$ (on calc)

get $x = a, b$

area $= \int_a^b \sin(\pi x) - (x^3 - 4x)\,dx$

c) area of cross-section =

S^2

S = side of square =

$(\sin(\pi x)) - (x^3 - 4x)$ ^{2}

volume $= \int_0^2 (\sin(\pi x) - (x^3 - 4x))^2\,dx$

d) Approximation of

area =

$$\sum \underbrace{((\sin(\pi x)) - (x^3 - 4x)) \cdot (3 - x)}_{\text{product of 2 factors}} \Delta x$$

$$= \int_0^2 (\sin(\pi x) - (x^3 - 4x)) \cdot (3 - x)\,dx$$

2)

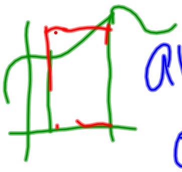
t	0	1	3	4	7	8	9
$L(t)$	120	156	176	126	150	80	0

L is twice-differentiable.

a) estimate an instantaneous rate of chg with average rate of change. [slope]

$$\text{rate} \approx \frac{L(7)-L(4)}{7-4} = \frac{150-126}{7-4} \text{ people per hour}$$

b) use TRAPEZOIDAL sum to est. average # of ppl in line from 0-4



average value of a f^n from a to b $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

$$\text{av val} = \frac{1}{4-0}\underbrace{\int_0^4 L(t)\,dt}$$

$A = \frac{1}{2}(b_1+b_2)h$

$$= \frac{1}{4}\left[\frac{1}{2}(120+156)(1) + \frac{1}{2}(156+176)(2) + \frac{1}{2}(176+126)(1)\right]$$

2)

t	0	1	3	4	7	8	9
$L(t)$	120	156	176	126	150	80	0

L is twice-differentiable.

c) For $[0,9]$, fewest # of times $L'(t)=0$, + reason.

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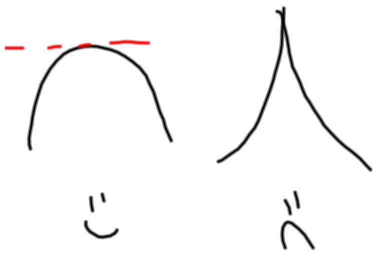
→ between $t=3$ and $t=4$ $\exists c$ with $L(c)=156$ (IVT)



between $t=3$ and $t=c$, the slope of the secant $=0$ $\therefore \exists k$ with $L'(k)=0$ between $t=3$ and c. (MVT)

-This point will be a rel. max.

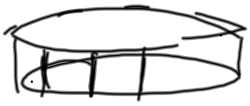
similarly between 3 & 7...
and between 4 and 8...

d) $r(t)=550te^{-t/2}$ tickets/hr

total tickets = starting tickets + total change in tickets

$$= \int_0^3 r(t)\,dt = 973\ldots$$

3) OIL $\frac{dV}{dt} = 2000$ cc/min



a) $r = 100$ cm & height $= .5$ cm $V = \pi r^2 h$

$\frac{dr}{dt} = 2.5$ cm/m

find $\frac{dh}{dt}$?

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi\left[2r\frac{dr}{dt}\cdot h + r^2 \cdot \frac{dh}{dt}\right]$$

$$2000 = \pi\left[2(100)(2.5)(.5) + (100)^2 \cdot \frac{dh}{dt}\right]$$

$$\frac{\frac{2000}{\pi} - (2)(100)(2.5)(.5)}{100^2} \frac{cm}{min} = \frac{dh}{dt}$$

b) $\frac{dV}{dt} = 2000 - 400\sqrt{t} = 0$

so $2000 = 400\sqrt{t}$

$5 = \sqrt{t}$

25 (min) $= t$

$\frac{d^2V}{dt^2} = -\frac{200}{\sqrt{t}}$ at $t = 25$, $\frac{d^2V}{dt^2} < 0$

so rel max

c) volume $= 60{,}000 + \int_0^{x} 2000 - 400\sqrt{t}\,dt$

so at $t = 25$

$$V = 60{,}000 + \int_0^{25} 2000 - 400\sqrt{t}\,dt$$